

Title	個体領域ノ個數條件ニ就イテ. II
Author(s)	平野, 次郎
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Osaka University

212. 個体領域、個數條件 = 就イテ。(II)

平野次郎 (阪大)

Lemma 2. $F_m(a, A) \leftrightarrow ({}_m x) A(x) \vee (\bar{A}(a) \& ({}_{m+1} x) A(x))$

□□ =

$$F_m(a, A): \prod_{i=1}^m (x_i) \left\{ \sum_{i,j}^{1,m} (x_i \neq x_j) \& \sum_{i=1}^m (x_i \neq a) \rightarrow \prod_{i=1}^m A(x_i) \right\}$$

$$({}_m x) A(x): \prod_{i=1}^m (x_i) \left\{ \sum_{i,j}^{1,m} (x_i \neq x_j) \rightarrow \prod_{i=1}^m A(x_i) \right\}$$

証明:

$$\text{先ツ } F_m(a, A) \rightarrow ({}_m x) A(x) \vee \bar{A}(a) \text{ ----- (i)}$$

$$\text{" } \rightarrow ({}_m x) A(x) \vee ({}_{m+1} x) A(x) \text{ ----- (ii)}$$

ヲ証明スル。

(i) ヲ次ノ如ク變形スル。

$$F_m(a, A) \& A(a) \rightarrow ({}_m x) A(x)$$

然ル =

$$A(a) \rightarrow \prod_{i=1}^m (x_i) \left\{ \prod_{i,j}^{1,m} (x_i = x_j) \vee \sum_{i=1}^m (x_i \neq a) \vee \prod_{i=1}^m A(x_i) \right\} \quad (1)$$

1) □ノ証明ハ次ノ如クスルベヨイ。

$m=1$ ノトキハ $A(a) \leftrightarrow (x)(x \neq a \vee A(x))$ 成立スル。

m ノトキ成立スルト假定スル。

$$A(a) \leftrightarrow (x_{m+1})(x_{m+1} \neq a \vee A(x_{m+1}))$$

$$x_{m+1} \neq a \vee A(x_{m+1}) \rightarrow \prod_{i=0}^{l, m+1} (x_i = x_j) \vee x_{m+1} = a \vee \prod_{i=1}^{m+1} A(x_i)$$

故 =

$$A(a) \rightarrow \prod_{i=1}^{m+1} (x_i) \left\{ \prod_{i,j}^{l, m+1} (x_i = x_j) \vee x_{m+1} = a \vee \prod_{i=1}^{m+1} A(x_i) \right\} \quad (1)$$

又

$$\begin{aligned} & \prod_{i,j}^{l, m} (x_i = x_j) \vee \sum (x_i \neq a) \vee \prod_{i=1}^m A(x_i) \\ & \rightarrow \prod_{i \neq j}^{l, m+1} (x_i = x_j) \vee \sum_{i=1}^m (x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i) \end{aligned}$$

故 =

$$A(a) \rightarrow \prod_{i=1}^{m+1} (x_i) \left\{ \prod_{i,j}^{l, m+1} (x_i = x_j) \vee \sum_{i=1}^m (x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i) \right\} \quad (2)$$

(1), (2) \exists \vee

$$A(a) \rightarrow \prod_{i=1}^{m+1} (x_i) \left\{ \prod_{i,j}^{l, m+1} (x_i = x_j) \vee \sum_{i=1}^{m+1} (x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i) \right\}$$

Q. E. D.

故 = . $F_m(a, A) \& A(a)$

$$\begin{aligned} & \rightarrow \prod_{i=1}^m (x_i) \left\{ \prod_{i,j}^{l, m} (x_i = x_j) \vee \left[\prod_{i=1}^m (x_i = a) \right. \right. \\ & \quad \left. \left. \& \sum_{i=1}^m (x_i \neq a) \right] \vee \prod_{i=1}^m A(x_i) \right\} \end{aligned}$$

$$\text{右辺ハ } \prod_{i=1}^m (x_i) \left\{ \prod_{i,j}^{1,m} (x_i = x_j) \vee \prod_{i=1}^m A(x_i) \right\} \quad \text{即チ } ({}_m x) A(x)$$

$$\text{故ニ } F_m(a, A) \& A(a) \longrightarrow ({}_m x) A(x)$$

$$(ii) \text{ハ } ({}_m x) A(x) \longrightarrow ({}_{m+1} x) A(x) \exists \vee$$

$$F_m(a, A) \longrightarrow ({}_{m+1} x) A(x)$$

ヲ証明スルベシ。

Lemma 1 $\exists \vee$

$$F_m(a, A) \longrightarrow A(x) \vee F_m(x, A)$$

$$\hookrightarrow \exists \vee$$

$$\begin{aligned} F_m(a, A) &\longrightarrow (x) \{ A(x) \vee F_m(x, A) \} \\ &\longrightarrow ({}_{m+1} x) A(x) \end{aligned}$$

次ニ逆ヲ証明スル。

$({}_m x) A(x) \longrightarrow F_m(a, A)$ ナルコトハ明カデアル故
 $\bar{A}(a) \& ({}_{m+1} x) A(x) \longrightarrow F_m(a, A)$ ヲ証明スル。

$$({}_{m+1} x) A(x) \longrightarrow A(a) \vee F_m(a, A)$$

$$\begin{aligned} \bar{A}(a) \& ({}_{m+1} x) A(x) &\longrightarrow \bar{A}(a) \vee (F_m(a, A) \& \bar{F}_m(a, A)) \\ &\qquad \qquad \qquad \& A(a) \vee \bar{F}_m(a, A) \end{aligned}$$

$$\longrightarrow (\bar{A}(a) \& A(a)) \vee F_m(a, A) \& \bar{A}(a) \vee \bar{F}_m(a, A)$$

$$\longrightarrow (\bar{A}(a) \& A(a)) \vee F_m(a, A)$$

$$\longrightarrow F_m(a, A)$$

$$\text{故ニ } ({}_m x) A(x) \vee \{ \bar{A}(a) \& ({}_{m+1} x) A(x) \} \longrightarrow F_m(a, A)$$

Q. E. D.

$$F_m(a, A) \leftrightarrow ({}_m x) A(x) \vee \{ \bar{A}(a) \& ({}_{m+1} x) A(x) \}$$

$$\wedge (x)(x \neq a \rightarrow A(x)) \infty (x)A(x) \vee \{\bar{A}(a) \& (2x)A(x)\}$$

ノ一; 擴張デアール. 之レ=就イテハ後=述ベタイト思フ。

次=、個數條件=就イテ、他ノ基本的ナ次ノ式ノ証明ヲ
試ヘル。

$$(x) \left(\sum_{i=1}^m (x \neq a_i) \rightarrow A(x) \right)$$

$$\infty (x)A(x)$$

$$\vee \left\{ \prod_{i=1}^m H_1(a_i; A) \& (2x)A(x) \right\}$$

$$\vee \left\{ \prod_{n_1, \dots, n_r}^{1, m} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \& (r+1)x)A(x) \right\}$$

$$\vee \left\{ \prod H_m(a_1, a_2, \dots, a_m; A) \& (m+1)x)A(x) \right\}$$

ココ=、

$$H_1(a_i; A) : \quad \bar{A}(a_i)$$

$$H_m(a_1, a_2, \dots, a_m; A) : \quad \sum_{i,j}^{1, m} (a_i \neq a_j) \& \sum_{i=1}^m \bar{A}(a_i).$$

$$\text{一般=} \quad H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) : \quad \sum_{i,j}^{1, r} (a_{n_i} \neq a_{n_j}) \& \sum_{i=1}^r \bar{A}(a_i)$$

証明: $m=1$ デ成立スル故, m デ成立スルモノト假定
スル。上ノ式=於テ、 $A(x)$ ノ代リ $x = a_{m+1} \vee A(x) : \mathcal{O}(x)$
ヲ代入スルトキハ、左辺ハ

$$(x) \left(\sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right) \quad \text{トナル。}$$

右辺へ $(x) \mathcal{O}(x)$

$$\vee \left\{ \prod_{i=1}^{l,m} H_i(a_i; \mathcal{O}) \& (2x) \mathcal{O}(x) \right\}$$

$$\vee \left\{ \prod_{n_1, \dots, n_r}^{l,m} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; \mathcal{O}) \& (r+1)x \mathcal{O}(x) \right\}$$

$$\vee \left\{ \prod H_m(a_1, a_2, \dots, a_m; \mathcal{O}) \& (m+1)x \mathcal{O}(x) \right\}$$

$$(x) \mathcal{O}(x) : (x) (x \neq a_{m+1} \rightarrow A(x))$$

$$\sim (x) A(x) \vee \left\{ \bar{A}(a_{m+1}) \& (2x) A(x) \right\}$$

$$\rightarrow (x) A(x) \vee \left\{ \prod_{i=1}^{l,m+1} H_i(a_i; A) \& (2x) A(x) \right\}$$

一般 =

$$\prod_{n_1, \dots, n_r}^{l,m} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; \mathcal{O}) \& (r+1)x \mathcal{O}(x)$$

$$\sim \prod_{n_1, \dots, n_r}^{l,m} \left\{ H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \& \sum_{i=1}^r (a_{n_i} \neq a_{m+1}) \right\}$$

$$\& F_{r+1}(a_{m+1}; A)$$

Lemma 2 \exists //

$$F_{r+1}(a_{m+1}, A) \sim (r+1)x A(x) \vee \left\{ \bar{A}(a_{m+1}) \& (r+2)x A(x) \right\}$$

故 = 上, 式は

$$\left[\prod_{n_1, \dots, n_r}^{l,m} \left\{ H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \& \sum_{i=1}^r (a_{n_i} \neq a_{m+1}) \right\} \right. \\ \left. \& (r+1)x A(x) \right]$$

$$\vee \left[\prod_{n_1, \dots, n_r}^{l, m} H_{r+1}(a_{n_1}, a_{n_2}, \dots, a_{n_r}, a_{m+1}; A) \& (r+2x) A(x) \right]$$

前項 = ツイテハ

$$\begin{aligned} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \& \sum_{i=1}^r (a_{n_i} \neq a_{m+1}) \\ \longrightarrow H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \\ \longrightarrow \prod_{n_1, \dots, n_r}^{l, m+1} H_r(a_{n_1}, \dots, a_{n_r}; A) \end{aligned}$$

$$\begin{aligned} \prod_{n_1, \dots, n_r}^{l, m} \{ H_r(a_{n_1}, \dots, a_{n_r}; A) \& \sum_{i=1}^r (a_{n_i} \neq a_{m+1}) \} \\ \longrightarrow \prod_{n_1, \dots, n_r}^{l, m+1} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \end{aligned}$$

後項 = ツイテハ

$$\begin{aligned} \prod_{n_1, \dots, n_r}^{l, m} H_{r+1}(a_{n_1}, a_{n_2}, \dots, a_{n_r}, a_{m+1}; A) \\ \longrightarrow \prod_{n_1, \dots, n_{r+1}}^{l, m+1} H_{r+1}(a_{n_1}, \dots, a_{n_{r+1}}; A) \end{aligned}$$

両者ヲ綜合シテ

$$\begin{aligned} \prod_{n_1, \dots, n_r}^{l, m} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}, \sigma) \& (r+1x) \sigma(x) \\ \longrightarrow \left[\prod_{n_1, \dots, n_r}^{l, m+1} H_r(a_{n_1}, a_{n_2}, \dots, a_{n_r}; A) \& (r+1x) A(x) \right] \\ \vee \left[\prod_{n_1, \dots, n_{r+1}}^{l, m+1} H_{r+1}(a_{n_1}, a_{n_2}, \dots, a_{n_{r+1}}; A) \& (r+2x) A(x) \right] \end{aligned}$$

$$\text{之 } \exists 1) \quad (x) \left(\sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right)$$

$$\rightarrow (x) A(x)$$

$$\vee \left\{ \prod_{i=1}^{1, m+1} H_1(a_i; A) \& (2x) A(x) \right\}$$

$$\vee \left\{ \prod_{n_1, \dots, n_r}^{1, m+1} H_r(a_{n_1}, \dots, a_{n_r}; A) \& (r+1)x A(x) \right\}$$

$$\vee \left\{ \prod H_{m+1}(a_1, \dots, a_{m+1}; A) \& (m+2)x A(x) \right\}$$

トナル、次ニコノ逆ヲ証明スル。

$$(r+1)x A(x) \rightarrow (x) \left(\sum_{i=1}^r (x \neq a_{n_i}) \rightarrow A(x) \right)$$

$$\vee \bar{H}_r(a_{n_1}, \dots, a_{n_r}; A)$$

$$(x) \left\{ \sum_{i=1}^r (x \neq a_{n_i}) \rightarrow A(x) \right\} \rightarrow (x) \left\{ \sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right\}$$

$$\text{故} = (r+1)x A(x) \rightarrow (x) \left\{ \sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right\}$$

$$\vee \bar{H}_r(a_{n_1}, \dots, a_{n_r}; A)$$

從ツテ

$$(r+1)x A(x) \rightarrow (x) \left\{ \sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right\}$$

$$\vee \sum_{n_1, \dots, n_r}^{1, m+1} \bar{H}_r(a_{n_1}, \dots, a_{n_r}; A)$$

故ニ前ト同様ニナル

$$\prod_{n_1, \dots, n_r}^{I, m+1} H_r(a_{n_1}, \dots, a_{n_r}; A) \& (r+1, x) A(x)$$

$$\rightarrow \prod_{n_1, \dots, n_r}^{I, m+1} H_r(a_{n_1}, \dots, a_{n_r}; A) \& (x) \left\{ \sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right\}$$

$$\vee \sum_{n_1, \dots, n_r}^{I, m+1} \bar{H}_r(a_{n_1}, \dots, a_{n_r}; A)$$

$$\exists 11 \rightarrow (x) \left(\sum_{i=1}^{m+1} (x \neq a_i) \rightarrow A(x) \right)$$

上、式ハ各 Disjunktionsglied ヲリ出ル故、逆ガ又成立スル。

勿論コノニ於ケル Induktion ハ前回同様有限的ノモノデアル。 — (未完) —